

# Resonant tunnelling in interacting 1D systems with an AC modulated gate

A.Komnik<sup>a</sup> and A.O.Gogolin<sup>b</sup>

<sup>a</sup>*Physikalisches Institut, Albert-Ludwigs-Universität, Hermann-Herder-Str. 3,  
D-79104 Freiburg, Germany*

<sup>b</sup>*Department of Mathematics, Imperial College London, 180 Queen's Gate, SW7  
1BZ London, United Kingdom*

---

## Abstract

We present an analysis of transport properties of a system consisting of two half-infinite interacting one-dimensional wires connected to a single fermionic site, the energy of which is subject to a periodic time modulation. Using the properties of the exactly solvable Toulouse point we derive an integral equation for the localised level Keldysh Green's function which governs the behaviour of the linear conductance. We investigate this equation numerically and analytically in various limits. The period-averaged conductance  $G$  displays a surprisingly rich behaviour depending on the parameters of the system. The most prominent feature is the emergence of an intermediate temperature regime at low frequencies, where  $G$  is proportional to the *line width* of the respective static conductance saturating at a non-universal frequency dependent value at lower temperatures.

*Key words:* Resonant tunnelling, Luttinger liquids, Carbon Nanotubes

*PACS:* 71.10.Pm, 73.63.Kv

---

## 1 Introduction

The future advances of the microelectronics depend crucially on the developments in the field of miniaturisation of electronic circuits. The ultimate

---

*Email address:* [komnik@uni-freiburg.de](mailto:komnik@uni-freiburg.de) (A.Komnik).

basis devices are single molecules or even atoms supplemented by three electrodes: source, drain and an additional gating electrode responsible for the current flow control. Such structures can, however, be so small that quantum effects have to be taken into account. The simplest possible realisation of such a device is a single fermionic level coupled to two metallic electrodes and subjected to an electrostatic interaction with a gate electrode. Such structures are also called single-state quantum dots. In the ideal situation of very small lateral device dimensions the contacting electrodes can be considered to be one-dimensional (1D). For instance, thin metallic single-wall carbon nanotubes (SWNTs) can play the role of such wirings. However, the low energy sector of 1D metals cannot be described by the conventional Fermi liquid theory, they are instead Luttinger liquids (LLs, for a recent review see e.g. [1]). Generally, the transport problem of a quantum dot coupled to LLs is not integrable [2]. Nevertheless, in some situations there exist unitary transformations which allow one to map the Hamiltonian of the system onto a diagonalisable quadratic one. In such situations all transport properties including the noise power spectra are directly accessible [3,4].

In addition to the static conductance properties future applications in integrated circuits demand also a detailed knowledge of the device response to AC currents and especially to an AC modulated gating. Thus far numerous studies of quantum dots under such conditions have been made [5,6,7]. However, all of them deal with quantum dot systems contacted by *non-interacting* electrodes. The purpose of this work is to gain initial insights into phenomena taking place in AC-modulated devices coupled to *interacting* LL electrodes. The outline of the paper is as follows. We begin by summarising the results for non-interacting systems in Section 2, where we also elaborate on the meaning of the low and high frequency approximations. We analyse the dynamic conductance for LL electrodes in Section 3.

## 2 Formulation of the problem and non-interacting results

The Hamiltonian under investigation,

$$H = H_K + H_t + H_C, \quad (1)$$

contains three parts. The first one,  $H_K$ , describes the dynamics of the resonant level and the electrons in the right/left (R/L) electrodes (we shall also call them leads),

$$H_K = \Delta(t)d^\dagger d + \sum_{i=R,L} H_0[\psi_i],$$

where the dynamic gating of the resonant level is described by explicitly time dependent level energy  $\Delta(t)$ . As long as the Kondo temperature is small, the spin effects are trivial for resonant tunnelling problem, therefore we work with a spinless model and briefly discuss the results for the spinful situations in Conclusions. The tunnelling between the resonant level and the electrodes with energy independent amplitudes  $\gamma_{R,L}$  is taken care of by the second term,

$$H_t = \sum_i \gamma_i [d^\dagger \psi_i(0) + \text{h.c.}] .$$

In (1),  $H_C$  stands for the electrostatic Coulomb interaction between the leads and the dot,

$$H_C = \lambda_C d^\dagger d \sum_i \psi_i^\dagger(0) \psi_i(0) .$$

In the non-interacting case, when  $H_0[\psi_i]$  describes free fermions in 1D and when  $\lambda_C = 0$  the non-linear  $I(V)$  can be shown in the static case to be given by the Meir-Wingreen formula [5],

$$I(V) = -\frac{e}{\pi} \int d\omega [f_L(\omega) - f_R(\omega)] \frac{\Gamma_L \Gamma_R}{\Gamma_R + \Gamma_L} \text{Im } D^R(\omega) , \quad (2)$$

where  $f_{L,R}(\omega) = n_F(\omega \mp eV/2)$  are the Fermi distribution functions in the leads and  $D^R(\omega)$  denotes the Fourier component of the retarded Green's function  $D^R(t, t')$  of the localised level.  $\Gamma_{R,L} = \pi \rho_0 \gamma_{R,L}^2$  are the dimensionless conductances of the contacts ( $\rho_0$  denotes the density of states in the leads). For the linear conductance one obtains then

$$G/G_0 = \frac{1}{4T} \int d\omega \cosh^{-2}(\omega/2T) T(\omega) \quad (3)$$

where  $T(\omega) = -\Gamma_L \Gamma_R \text{Im } D^R(\omega) / (\Gamma_R + \Gamma_L)$  is the transmission coefficient of the electrons through the double constriction and  $G_0 = e^2/h$  denotes the conductance quantum. In the static case  $D^R(t, t')$  depends only on the difference  $t - t'$  while in case of the dynamic gating it depends on  $t - t'$  as well as  $t + t'$ . In such situations one is usually interested in the current (or conductance) averaged over the period of the gate voltage oscillations. The non-interacting situation has been discussed in detail in Ref.[6]. Here we are interested in the linear conductance rather than in the full  $I(V)$ . Therefore we recapitulate and somewhat extend the results of Ref.[6] in order to be able to compare to the interacting results we're presenting in the next Section. In this situation  $D^R(t, t')$  can be solved for exactly leading to [7]

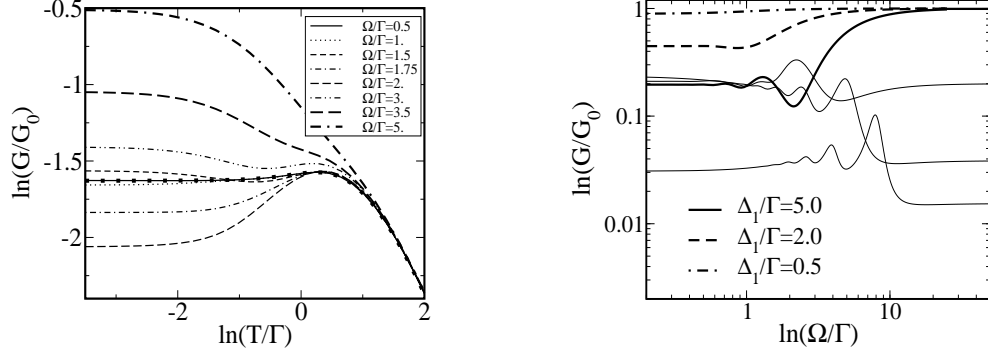


Fig. 1. *Left panel:* Temperature dependence of the period averaged linear conductance for different gate oscillation frequencies and  $\Delta_1/\Gamma = 5$ ,  $\Delta_0 = 0$ . The thick dotted line (not included into the legend) represents the result of the adiabatic approximation. *Right panel:* Thick lines: frequency dependence of the asymptotic linear conductance at  $T = 0$  for different values of  $\Delta_1$ ,  $\Delta_0 = 0$ . Thin lines represent the cases of nonzero  $\Delta_0/\Gamma = 2, 5, 8$  and  $\Delta_1/\Gamma = 5$  (from above). Notice the double logarithmic scale.

$$D^R(t, t') = -i\Theta(t - t') \exp \left[ -i \int_{t'}^t d\tau \Delta(\tau) - \frac{1}{2}(\Gamma_L + \Gamma_R)(t - t') \right]. \quad (4)$$

Assuming the gating is harmonic,  $\Delta(t) = \Delta_0 + \Delta_1 \cos(\Omega t)$ , one can easily perform the time integrations and the averaging over the period  $2\pi/\Omega$  with respect to  $t + t'$ . Then, using the Bessel  $J_0$  function one obtains [6]:

$$\bar{D}^R(t - t') = -i\Theta(t - t') e^{[-i\Delta_0 - (\Gamma_L + \Gamma_R)/2](t - t')} J_0 \left\{ \frac{2\Delta_1}{\Omega} \sin[\Omega(t - t')/2] \right\}.$$

As a result we obtain for the imaginary part [from now on we measure all energies in units of  $(\Gamma_R + \Gamma_L)/2$ ]

$$\text{Im} \bar{D}^R(\omega) = - \sum_{n=-\infty}^{\infty} \frac{J_n^2(\Delta_1/\Omega)}{(\omega - \Delta_0 + n\Omega)^2 + 1}. \quad (5)$$

Insertion of this relation into (3) yields then the following analytic formula for the linear conductance

$$G/G_0 = \frac{4\Gamma_R\Gamma_L}{(\Gamma_R + \Gamma_L)^2} \frac{1}{2\pi T} \sum_n J_n^2(\Delta_1/\Omega) \text{Re} \left\{ \Psi' \left[ \frac{1}{2} + \frac{1 + i(n\Omega - \Delta_0)}{2\pi T} \right] \right\} \quad (6)$$

At least in the non-interacting situation the asymmetry effects are trivial and are only reflected by the corresponding prefactor, so that we restrict our analysis to  $\Gamma_R = \Gamma_L$ . At  $T \gg \Gamma$ , the conductance ceases to depend on either  $\Omega$  or  $\Delta_i$  ( $i = 1, 2$ ), and is given by  $G/G_0 \approx \pi/4T$ . In the opposite limit of low temperatures the main feature of  $G$  is generic: it saturates as  $T$  approaches zero.

However, the saturation values are not the same at different oscillation frequencies. For very small frequencies and  $\Delta_0 \lesssim \Delta_1$ ,  $G(0)/G_0 \approx 1/\Delta_1$  whereas for  $\Delta_0 \gg \Delta_1$ , the conductance decays as  $\sim \Delta_0^{-2}$ , see Fig.1. From the physical point of view it can be understood as follows. In the static situation due to finite tunnelling amplitude the localised level and the leads hybridise, so that the spectral function of the dot state has a finite width  $\sim \Gamma$ . Then, as long as  $\Delta_0 \lesssim \Delta_1$ , the level spends a fraction  $\nu = \Gamma/\Delta_1$  of the oscillation period within ‘reach’ of the electrodes. That’s why the averaged conductance is proportional to  $\nu$ . On the other hand, for  $\Delta_0 \gtrsim \Delta_1$  the overlap of two areas cannot ever become maximal. That leads to a fast decay of the averaged  $G$  as a function of  $\Delta_0$ . In the opposite case of very high frequencies the average time, which an electron needs to be transferred through the constriction,  $1/\Gamma$ , is much larger than the oscillation period, so that the particles only see the level being ‘frozen’ at its average energy  $\Delta_0$ , which means that  $G/G_0 \approx (1 + \Delta_0^2)^{-1}$  in that limit. The crossover between large and small frequencies is highly non-monotonous, see Fig.1. There are local maxima at  $\Omega = \Delta_0/(2n + 1)$  and minima at  $\Omega = \Delta_0/2n$  ( $n$  is a natural number), which correspond to absorption of multiple energy quanta.

Of course, all these results can be recovered using the corresponding asymptotic expansions of the exact result (6), see Appendix. Moreover, the asymptotic behaviour of the averaged conductance can be extracted from the static level Green’s function without knowledge of the exact dynamic one (4). At very low frequencies  $\Omega \ll \Gamma$  the system can be regarded to be in equilibrium with respect to the oscillations at any time and the obvious approximation is thus given by (2) with the period-averaged transmission coefficient  $T(\omega, t) = [1 + (\omega - \Delta(t))^2]^{-1}$  (we are still dealing with the symmetric situation  $\Gamma_R = \Gamma_L$ )

$$\bar{T}(\omega) = \frac{\Omega}{2\pi} \int_0^{2\pi/\Omega} dt T(\omega, t) = \text{Re} \left\{ [1 + i(\omega - \Delta_0)]^2 + \Delta_1^2 \right\}^{-1/2}. \quad (7)$$

The results of this *adiabatic* approximation are depicted in Fig.1 and describes very well the behaviour of  $G$  for frequencies up to  $\Gamma$ . We wish to stress here that  $\Omega \rightarrow 0$  limit is non-trivial and is *not* equivalent to simply taking  $\Delta_1 = 0$ . Since the usual experimental values of  $\Gamma$  range between 0.1 and 1  $\mu\text{eV}$ , the static approximation is expected to be valid to fairly high frequencies of the order of several MHz. The opposite limit of  $\Omega \gg \Gamma$  is even simpler. In that situation one can use the formula for the static case with  $\Delta(t) \rightarrow \Delta_0$ . This approach we refer to as *anti-adiabatic* approximation.

### 3 Interacting leads

It has been shown in a number of recent works that the resonant tunnelling between correlated electrodes is completely different from the non-interacting case [8,9,10,3,11,4,12]. As in the non-interacting situation, we start with the Hamiltonian (1) with  $\lambda_C \neq 0$ . However, contrary to the previous case, we model the leads by half-infinite LLs. It turns out that in the static case the problem can be solved exactly at the so-called Toulouse point, when the LL interaction parameter  $g = 1/2$  and  $\lambda_C = 2\pi$  [3]. Generally, the transport properties can be extracted in two different ways, either using the equations of motion method [3], or via Keldysh diagram technique [4]. The latter approach is, however, considerably easier to apply in the case of a dynamic resonant level. After a unitary transformation, refermionization and introduction of the Majorana components of the new fermionic degrees of freedom,  $\xi$ ,  $\eta$ , and those for the dot level,  $a$ ,  $b$ , the resulting Hamiltonian can be written in the form  $H = H_0 + H_t$ , where (see [3,4] for details of the mapping; we use the same notation)

$$H_0 = i\Delta(t)ab + i \int dx [\eta(x)\partial_x\eta(x) + \xi(x)\partial_x\xi(x) + V\xi(x)\eta(x)], \quad (8)$$

and  $H_t = i\gamma b\xi(0)$ , while the current operator is given by  $J = -i\gamma b\eta(0)$ . Calculating the average of this operator using the  $S$  matrix containing the  $H_t$  coupling, one can derive the following expression for the full (time-dependent) non-linear  $I - V$  of the system,

$$I(V) = \frac{e}{8\pi}\Gamma \int d\omega [n_F(\omega + eV) - n_F(\omega - eV)] \text{Im} D_{bb}^R(\omega, t), \quad (9)$$

where  $\Gamma = \gamma^2$  and  $D_{bb}^R(\omega, t)$  is the Fourier component of the retarded Green's function (GF) of the  $b$  Majoranas with respect to the difference of time arguments,

$$D_{bb}^R(\omega, t = \tau + \tau') = \int d(\tau - \tau') \exp[i\omega(\tau - \tau')] D_{bb}^R(\tau, \tau'). \quad (10)$$

As in the non-interacting case, the linear conductance turns out to be given by the same formula (3). The meaning of the transmission coefficient  $T(\omega)$ , of course, changes. Now it is related to the probability for the 'new' fermions (see Ref.[3]) to be scattered off the resonant level. According to Ref. [4] in the static case one obtains

$$T(\omega) = \omega^2 / [(\omega^2 - \Delta^2)^2 + \omega^2], \quad (11)$$

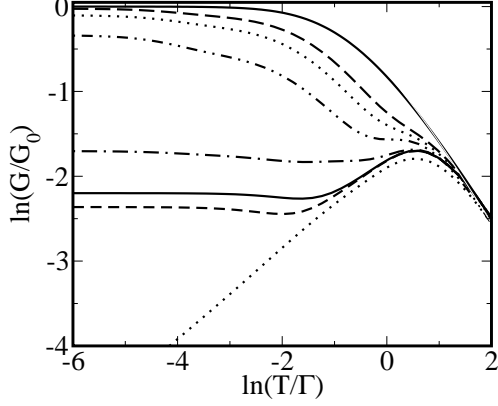


Fig. 2. Temperature dependence of the averaged conductance in the interacting case for different gate frequencies  $\Omega$  and  $\Delta_0 = 0$ . The uppermost curve represents the result of the anti-adiabatic approximation while the lowermost curve is the result of the adiabatic approximation. The curves in between correspond to  $\Omega/\Gamma = 4, 3.5, 3, 1, 0.1$  and  $0.05$  (from above). Notice the double logarithmic scale.

while for dynamic gating the effective  $T(\omega)$  has to be found via solution of corresponding Dyson equation for the retarded GF,

$$D_{bb}^R(t, t') = D_{bb}^{(0)R}(t, t') + \Gamma \int d\tau d\tau' D_{bb}^{(0)R}(t, \tau) G_{\xi\xi}^{(0)R}(\tau - \tau') D_{bb}^R(\tau, t'), \quad (12)$$

where  $G_{\xi\xi}^{(0)R}(\tau - \tau') = -i\delta(\tau - \tau')/2$  is the retarded GF of the lead Majoranas for  $\gamma = 0$  and

$$D_{bb}^{(0)R}(t, t') = -i\Theta(t - t') \cos\left[\int_{t'}^t d\tau \Delta(\tau)\right] \quad (13)$$

is the bare retarded GF of the resonant level Majoranas. Unfortunately we could not solve Eq.(12) analytically. Before we present numerical results, let us discuss the limiting cases of low and high  $\Omega$ . As expected from the analysis of the non-interacting case, these limiting cases should be reliably described by the adiabatic and the anti-adiabatic approximations, respectively. Let us first investigate the most interesting case of zero off-set  $\Delta_0 = 0$ . The  $\Omega \rightarrow \infty$  limiting case is trivial and the conductance is given by the corresponding static result with  $\Delta_{0,1} = 0$ , see the uppermost curve in Fig.2. In the case of slow oscillations we resort to the adiabatic approximation, which can be implemented in the same way as in the non-interacting situation, namely by averaging the corresponding static transmission coefficient (11) over the oscillation period:

$$\bar{T}(\omega) = \left| \frac{2\omega}{\Delta_1^2} \text{Im} \left\{ [2(\omega^2 - i\omega)/\Delta_1^2 - 1]^2 - 1 \right\}^{-1/2} \right|. \quad (14)$$

Evaluation of the linear conductance using this approximation results in the lowest curve in Fig. 2. The main feature is that  $G$  vanishes at zero temperature as  $\sim \sqrt{T/\Gamma}$ . To explain that we can proceed along the same line of reasoning as in the non-interacting case. It is reasonable to assume that in this particular situation the dominant contribution to  $G$  is generated near the resonance, when  $\Delta$  is aligned with the chemical potentials in the leads. The averaged conductance of the structure is then proportional to the ratio of the effective level width  $w(T)$  to the amplitude  $\Delta_1$  of the oscillation  $G \sim w(T)/\Delta_1$ . As we know from the solution of the symmetric static case at  $g = 1/2$ , the level width vanishes as a square root of temperature [10,3]. That is why we find the same power-law for  $G(T)$ . Note that this is very much different from the non-interacting case where, of course,  $w(T)$  does not depend on temperature. In order to obtain information about  $G$  at intermediate frequencies one has to solve equation (12) and extract the transmission coefficient numerically. The outcome of this approach is shown in Fig. 2. As can be seen from this figure, there is an important qualitative correction to the adiabatic approximation. Namely, the exact  $G$  saturates as  $T \rightarrow 0$  at a finite  $\Omega$ -dependent value:  $G(T = 0) \sim \sqrt{\omega}$ . The on-set of this saturation takes place at  $T^* \sim \Omega$ . At temperatures higher than  $T^*$  the system finds itself in the low frequency limit and  $G$  follows the square root power-law found in the static approximation. On the contrary, below  $T^*$  the constriction can be considered as being in the high frequency limit, where the conductance is expected to saturate. The resonant constellation  $\Delta(t) \sim 0$  ceases to generate the dominant contribution becoming too narrow in comparison to the off-resonant positions of the dot level, when the effective width is known to saturate [10,3].

The results for the ‘immersed’ level, when  $\Delta_0 < \Delta_1$  are very similar, see Fig. 3. Again, the conductance curves saturate at temperatures  $T \ll \Omega$ . However, the (anti-)adiabatic approximation as well as the dynamic curves in the intermediate regime show different power-laws: instead the square root in the resonant ( $\Delta_0 = 0$ ) situation the conductance decays toward low temperatures quadratically. This power-law originates in the different transport mechanism: now the most of the conductance does not come from the resonant transmission, which cannot be achieved any more, but from the off-resonance processes which show quadratic temperature behaviour. Moreover, in the immersed case the adiabatic approximation yields higher conductances than the anti-adiabatic ones. The asymptotic conductance behaviour at  $T = 0$  shows another interesting feature. It is maximal when the oscillation frequency matches the off-set energy  $\Delta_0$ . In that situation the tunnelling electron can absorb an energy quantum from the gate and get transmitted through the system as if there is no energy off-set.



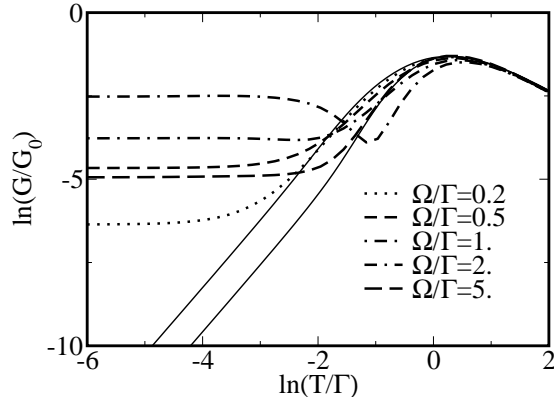


Fig. 3. Temperature dependence of the averaged conductance in the interacting case for different gate frequencies  $\Omega$  and  $\Delta_0/\Gamma = 2.$ ,  $\Delta_1/\Gamma = 1.$ . The upper solid line represents the result of the adiabatic approximation while the lower one corresponds to the anti-adiabatic approximation. Notice the double logarithmic scale.

## 4 Conclusions

We have analysed transport properties of a single site quantum dot coupled to two LL electrodes in the case of an AC modulated gate. Using a unitary transformation and reformation we were able to recast the Hamiltonian into a quadratic form at a special interaction strength  $g = 1/2$ . This allowed for an exact calculation of the experimentally relevant period averaged conductance. Its temperature dependence turns out to possess a rich behaviour, showing two cross-over points. Upon lowering the temperature for  $T \gg \Gamma$  the effect of the AC modulation is not visible and  $G(T)$  follows the same power-law as in the static case  $\sim \Gamma/T$  no matter how large is the off-set energy  $\Delta_0$ . In the case of  $\Delta_0 = 0$  and for oscillation frequencies  $\Omega \gg \Gamma$  the averaged conductance saturates at a value larger than  $G(T = \Gamma)$ . However, as soon as  $\Omega$  becomes smaller than  $\Gamma$ , the conductance vanishes first as a square root of temperature in the interval between  $T = \Omega$  and  $T = \Gamma$  and after that saturates at a finite value. The saturation toward low temperature is the most prominent result of a dynamic gating and has a potential to obscure the resonant tunnelling physics occurring in strictly static set-ups. Nevertheless, an AC modulation experiment has additional advantages against the static one - it enables an extraction of the temperature dependence of the resonance peak width from the information about  $G(T)$  in the intermediate temperature regime  $\Omega < T < \Gamma$ .

Our findings are, strictly speaking, restricted to the spinless  $g = 1/2$  case. Based on this results and the achieved understanding of various conductance mechanisms, we can however speculate about the power-laws that will emerge in the general- $g$  case as well as in set-ups involving electrons with spin (so long as Kondo physics is not important) and SWNTs. For the spinless system the most important regimes are the following: (i) the high temperature limit  $G(T) \sim T^{2g-2}$ ; (ii) the intermediate regime  $G(T) \sim T^{1-g}$ ; (iii) saturation

to a frequency dependent value at lowest temperatures. The results for the resonant tunnelling between two spinful LLs and SWNTs are obtained by a substitution  $g \rightarrow (g + 1)/2$  for a system with spin and  $g \rightarrow (K + 3)/4$  for an SWNT, where  $K$  is the nanotube Luttinger parameter. We shall present the detailed analysis of the general- $g$  case in a separate publication [13].

To summarise, the dynamic resonant conductance for LL leads is markedly different from that for non-interacting contacts. It possesses various distinctive features which are, in principle, verifiable experimentally. In our view a comparison of static and dynamic (AC-modulated) conductance measurements on the same system would be especially valuable.

## A Adiabatic approximation

In this Appendix, we elaborate on the exact formula (6) for the conductance in the non-interacting case at  $T = 0$ . Expanding (6) in  $T$  we obtain:

$$G/G_0 = \sum_{n=-\infty}^{\infty} \frac{J_n^2(\Delta_1/\Omega)}{(n\Omega - \Delta_0)^2 + 1} . \quad (\text{A.1})$$

First of all, it is useful to notice that  $G/G_0$ , as given by formula (A.1), is always smaller than 1. This immediately follows from the well known identity (see, e.g., [14])

$$\sum_{n=-\infty}^{\infty} J_n^2(z) = 1 .$$

The anti-adiabatic case ( $\Omega \gg \Delta_1$ ) is trivial. For  $\Delta_0 = 0$  the perfect conductance is recovered. Expanding the Bessel functions, we find the next-to-leading correction as

$$G/G_0 = 1 - \frac{1}{2} \frac{\Omega^2}{\Omega^2 + 1} \left( \frac{\Delta_1}{\Omega} \right)^2 + \dots$$

Note that the perturbative correction in  $\Delta_1$  is converging for all  $\Omega$ .

Next we consider the adiabatic case  $z = \Omega/\Delta_1 \ll 1$ . This turns out to be more involved. Using another well known identity (see [14]),

$$\sum_{n=-\infty}^{\infty} J_n^2(z) e^{-in\phi} = J_0[2z \sin(\phi/2)] ,$$

we re-write formula (A.1) as

$$G/G_0 = \frac{1}{\pi} \text{Re} \int_0^\pi d\phi J_0[2z \sin(\phi/2)] f(\phi) , \quad (\text{A.2})$$

with

$$f(\phi) = \sum_{n=-\infty}^{\infty} \frac{e^{in\phi}}{(n\Omega - \Delta_0)^2 + 1} .$$

This sum can be evaluated by using Poisson summation formula with the result

$$f(\phi) = \frac{\pi}{\Omega} \frac{e^{(\pi-\phi)(1-i\Delta_0)/\Omega}}{2 \sinh[\pi(1-i\Delta_0)/\Omega]} + \frac{\pi}{\Omega} \frac{e^{(\phi-\pi)(1+i\Delta_0)/\Omega}}{2 \sinh[\pi(1+i\Delta_0)/\Omega]} .$$

When substituting the above expression into formula (A.2) for conductance we observe that, in the  $\Omega \rightarrow 0$  limit, the  $\phi$ -integral converges in a narrow interval  $\phi \sim \Omega$ . Rescaling  $\phi = \Omega\theta$  and taking  $\Omega \rightarrow 0$  limit we find

$$\lim_{\Omega \rightarrow 0} G/G_0 = \text{Re} \int_0^\infty d\theta e^{-(1-i\Delta_0)\theta} J_0(\Delta_1\theta) = \text{Re} \frac{1}{\sqrt{(1-i\Delta_0)^2 + \Delta_1^2}} . \quad (\text{A.3})$$

This result justifies the use of the adiabatic approximation adopted in the main text.

## References

- [1] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelick, *Bosonization and Strongly Correlated Systems*, (Cambridge University Press, Cambridge, UK).
- [2] A. Furusaki, Phys. Rev. B **57**, 7141 (1998).
- [3] A. Komnik and A. O. Gogolin, Phys. Rev. Lett. **90**, 246403 (2003).
- [4] A. Komnik and A. O. Gogolin, Phys. Rev. B **68**, 235323 (2003).
- [5] Y. Meir and N. S. Wingreen, Phys. Rev. Lett. **68**, 2512 (1992).
- [6] A.-P. Jauho, N. S. Wingreen, and Y. Meir, Phys. Rev. B **50**, 5528 (1994).
- [7] D. C. Langreth and P. Nordlander, Phys. Rev. B **43**, 2541 (1991).
- [8] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).

- [9] A. Furusaki and N. Nagaosa, Phys. Rev. B **47**, 3827 (1993).
- [10] Yu. V. Nazarov and L. I. Glazman, Phys. Rev. Lett. **91** 126804 (2003).
- [11] D. G. Polyakov and I. V. Gornyi, Phys. Rev. B **68** 035421 (2003).
- [12] S. Hügler and R. Egger, cond-mat/0304158.
- [13] A. Komnik and A.O. Gogolin (unpublished).
- [14] G.N. Watson, *A Treatise on the Theory of the Bessel Functions*, Cambridge (1992).